Integration- An Intuitive Approach

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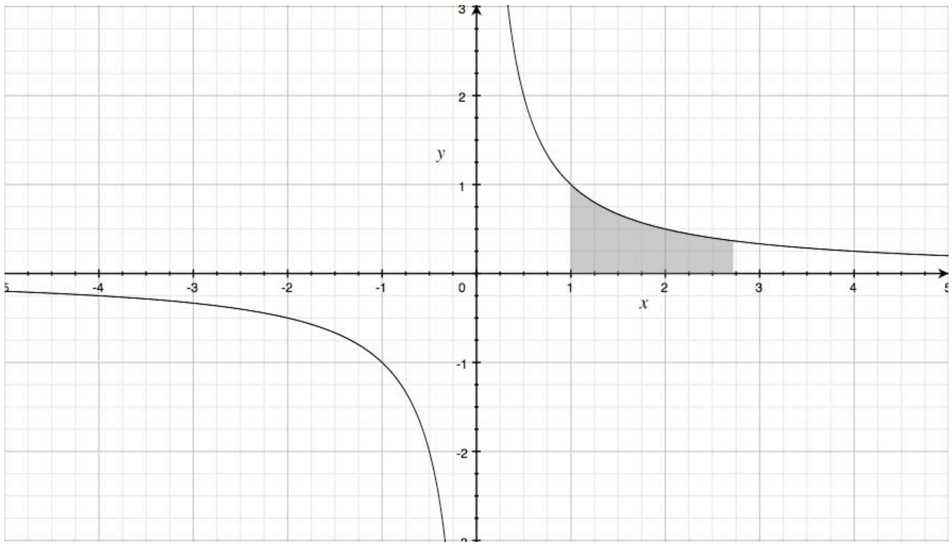
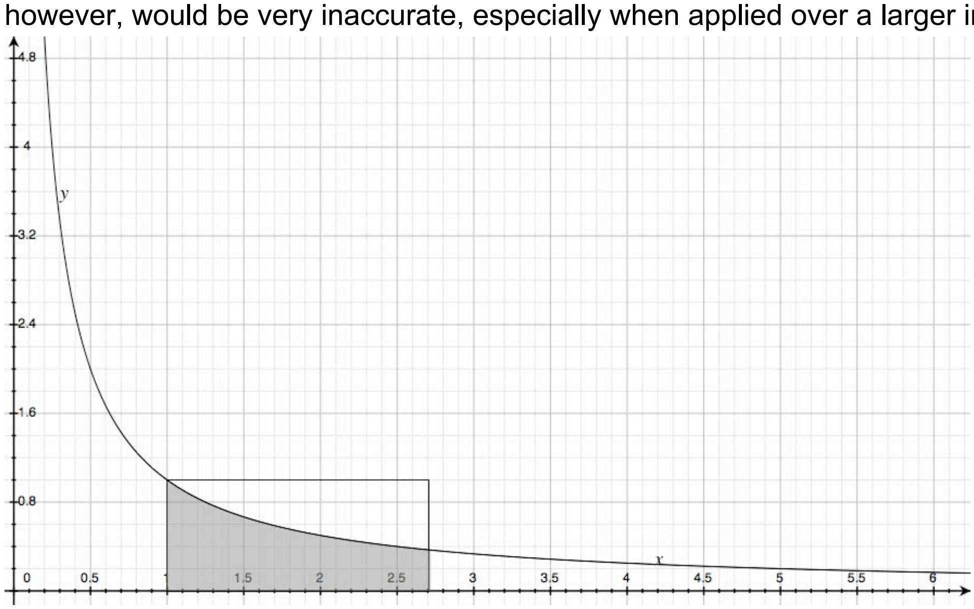


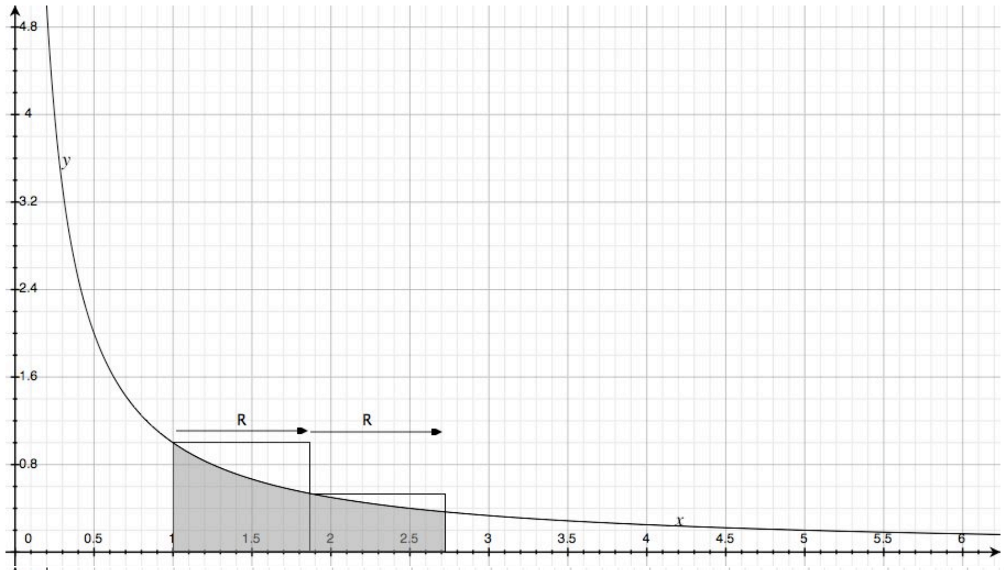
Riemann Sums

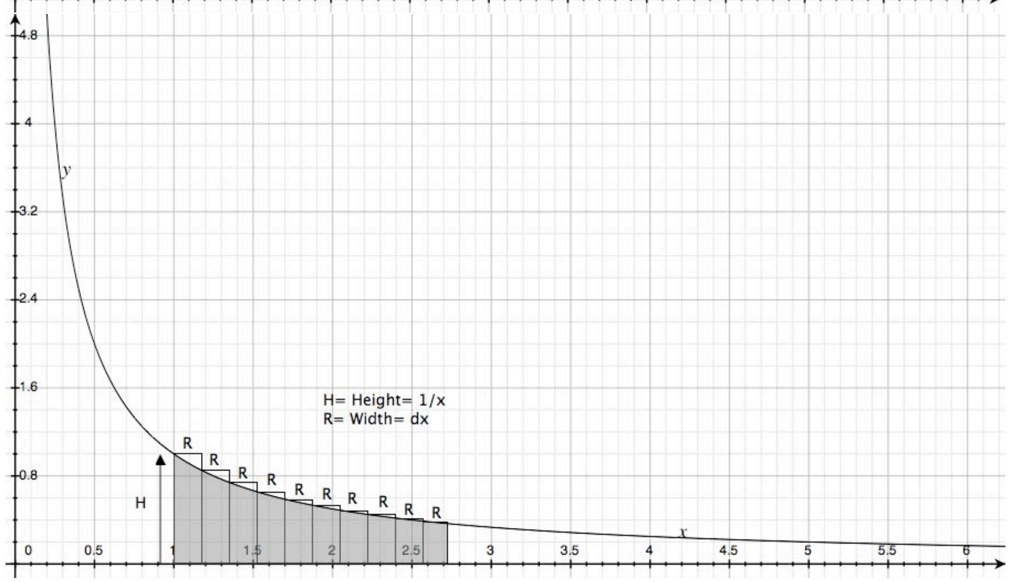
On any graph, there is an amount of area between the x-axis and the curve. The area under this curve can be measured.The area on the graph of f(x)=1/x from 1 to e is represented on the above graph.

Now imagine that the whole of the area is encompassed by a rectangle, with a height that matched the height of f(x) at its left corner (1), and a width of e-1 (the width of the interval). This rectangle could be used to approximate the are under the curve of f(x). The approximation, however, would be very inaccurate, especially when applied over a larger interval.



If we divide the section into two rectangles, each with a height of f(x) at its left corner, we would end up with a slightly more accurate approximation of the area.





The more rectangles the section is divided into, the better the approximation of area. This is a Reimman Sum. More specifically, it is a left, upper Reimann Sum (denoted by where the rectangle touches the curve). A Riemann sum is a method of approximating area under a curve. The formula for the area of a rectangle is length times width. The summation of those areas would therefore be the sum of the individual lengths and widths. If all rectangles were the same width however, the formula could be simplified.

The height- On the interval [a,b], (b-a) represents the width, divided by n, the number of rectangles, gives us the width of each rectangle. Starting at a, if you add one width every time to reach the next rectangle, and take the function of those values, you end up with the sum of the heights.

$$\frac{(b\text{-}a)}{n}\sum_{i=1}^{n} f\left(a+\frac{(b\text{-}a)i}{n}\right)$$

The width- The length of the interval, divided by the number of rectangles

$$\frac{(e\text{-}1)}{10}\sum_{i=1}^{10}\frac{1}{\left(1+\frac{(e\text{-}1)i}{10}\right)} = \frac{(e\text{-}1)}{10}\left(\frac{1}{1+\frac{(e\text{-}1)}{10}}+\frac{1}{1+\frac{(e\text{-}1)2}{10}}+\frac{1}{1+\frac{(e\text{-}1)3}{10}}\cdots\cdots\frac{1}{1+\frac{(e\text{-}1)10}{10}}\right)=.9478$$

Using our 1/x equation with ten rectangles

Now, applying the ideas of calculus, we judge that if there were an infinite number if rectangles, each infinitely small, we would end up with an exact amount for the area under any given curve.

$$\frac{\lim}{n\text{->}\infty}\left(\frac{(b\text{-}a)}{n}\sum_{i=1}^{n} f\left(a+\frac{(b\text{-}a)i}{n}\right)\right) \qquad =\int_a^b f(x)dx$$

This is the definition of an Integral.

We can also think of an integral as an antiderivative. If you take the derivative of a function, you get the slope of the line at a point. If you take the integral of a function, however, you get the area under the curve of the function. You can think of this in terms of adding dimensions. I you have a point (derivative) and add a dimension, you have a line. If you add an infinite number of lines, you get a plane. This plane is area. The functions are inverses. If you have the Integral from a to b of a curve, you have the antiderivative of that curve. If you take the derivative of that, you end up with you original function, from a to b.

The Integral of a function on an interval, or the Definite Integral is simply the integral of the function up to the end point, minus the integral up to the beggining point.

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b)-F(a)$$

This transfers into the fact that the derivative of a definite integral can be evaluated as the change in the amount of the function.

$$\frac{\partial}{\partial x}\left[\int_a^b f(x)dx\right] = \frac{\partial}{\partial x}[F(b)-F(a)]$$

$$= \frac{\partial}{\partial x}\left[\int_a^b f(x)dx\right] = f(b)-f(a)$$

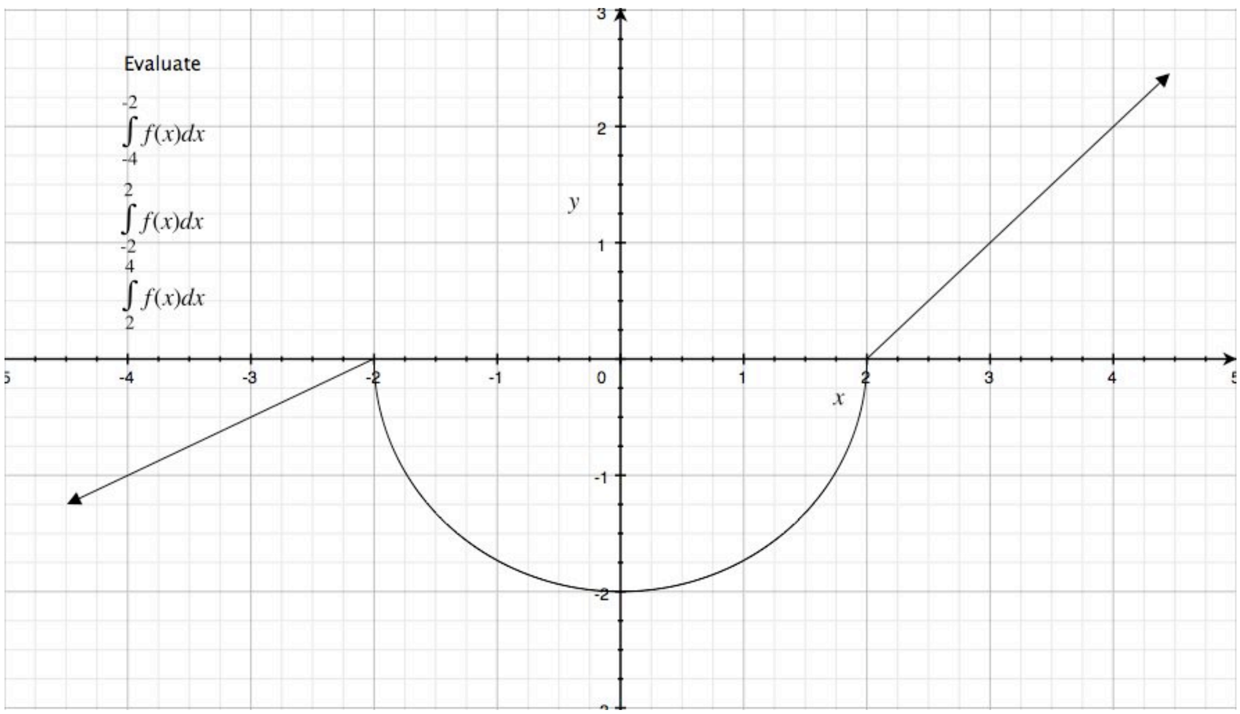
<u>Rules for definite integrals</u>

$$\int_a^a f(x)dx = 0 \qquad \int_a^b \{f(x) \pm g(x)\}dx = \int_a^b f(x)dx \pm \int_a^b g(x)$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx \qquad \int_a^b kf(x)dx = k\int_a^c f(x)dx$$

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

[Sect 4.3]



Consider the Equation f that is continuous on the interval [-5,5], and for which

$$\int_{0}^{5} f(x)dx = 4$$

Evaluate each integral

$$\int_{0}^{5} f[(x)+2]dx$$

$$\int_{-2}^{3} f[(x+2)]dx$$

$$\int_{-5}^{5} f[(x)]dx \qquad \text{f is even}$$

$$\int_{-5}^{5} f[(x)]dx \qquad \text{f is odd}$$

Evaluate the integrals

$$\int_{1}^{e} \left(\frac{1}{x}\right)dx \qquad 2\int_{1}^{16} \frac{x^2}{\left(\frac{1}{3}\right)}dx$$

Sketch the region given by the Integral. Then evaluate geometrically. (a>0)

$$\int_{0}^{3} 4dx \qquad \int_{0}^{8} (8-x)dx \qquad \int_{-3}^{3} \left(\sqrt{9-x^2}\right)dx \qquad \int_{-a}^{a} (a-|x|)dx$$

Given $\int_{-1}^{1} (x)dx = 0$ and $\int_{0}^{1} (x)dx = 5$ evaluate

$$\int_{-1}^{0} f(x)dx \qquad \int_{0}^{1} f(x)dx - \int_{-1}^{0} f(x)dx$$

$$\int_{-1}^{1} 3f(x)dx \qquad \int_{0}^{1} 3f(x)dx$$